

# Gas Jet Characterization via interferometric phase recovery

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## Abstract

In the following report, I'm going to review, in details, a well-known technique to characterize the spacial profile of a **gas jet** via **phase reconstruction** of a light **interferometric pattern**. This pattern can be, simply, obtained with a **Mach-Zehnder interferometer**, in which the gas jet is present in one of the two optical path.

The paper is organized as follows:

- First, the simple physics behind phase recovery will be discussed, by focusing on the gas jet problem and the **interferometric fringes** pattern production.
- Then, some **Matlab simulations**, that I have implemented, will be presented and peculiar phase profile, will be recovered from the simulated fringes.
- At the end, I'm going to present the **experimental setup**, used for real gas jet profile characterization, and the experimental results achieved with the phase recovery code.

The interferometric fringes recovery Matlab program is available at the following git repository.

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## 1. Interferometric fringes distortion due to 3D gas jet

### 1.1 Two waves interference

Consider two monochromatic waves, with wave vectors  $\mathbf{k}_1$ ,  $\mathbf{k}_2$  and frequency  $\omega$ , that overlap in a certain spacial region.

$$\mathbf{E}_1(\mathbf{r}) = \mathbf{E}_{01} \cos(\mathbf{k}_1 \mathbf{r} - \omega t + \varphi_0) \quad (1)$$

$$\mathbf{E}_2(\mathbf{r}) = \mathbf{E}_{02} \cos(\mathbf{k}_2 \mathbf{r} - \omega t + \varphi(x, y)) \quad (2)$$

Where we're assuming that the (1) has a flat phase-front, while (2), for yet unspecified reason, has a distorted phase front due to  $\varphi(x, y)$ .

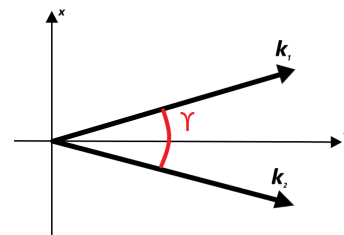
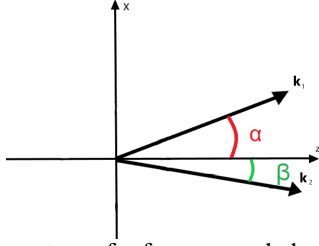


Figure 1. Electromagnetic fields wave vectors

For simplicity, we explicit the direction of the two vectors, fixing the more comfortable system of reference, as well as the polarization of the electric fields, assumed among y.



**Figure 2.** New system of reference angle between  $[-\pi, +\pi]$

At this point we can rewrite the two electric fields in a scalar form and look in  $(x, y, 0)$  plane.

$$\begin{aligned} E_{1y}(x, y, 0) &= E_{01} \cos(k \sin(\alpha)x - \omega t + \varphi_0) \\ E_{2y}(x, y, 0) &= E_{02} \cos(k \sin(\beta)x - \omega t + \varphi(x, y)) \end{aligned}$$

In a "small-angles" approximation:

$$E_{1y}(x, y, 0) = E_{01} \cos(k\alpha x - \omega t + \varphi_0) \quad (3)$$

$$E_{2y}(x, y, 0) = E_{02} \cos(k\beta x - \omega t + \varphi(x, y)) \quad (4)$$

$$(5)$$

Since the two field are spatially overlapped, they make interference producing a final field  $E_{tot}$ :

$$E_{tot}(x, y, 0) = E_{1y}(x, y, 0) + E_{2y}(x, y, 0) \quad (6)$$

Now, remembering the following trigonometric rule:

$$\cos(\Gamma) + \cos(\Omega) = 2\cos\left(\frac{\Gamma + \Omega}{2}\right)\cos\left(\frac{\Gamma - \Omega}{2}\right)$$

We can, easily, achieve:

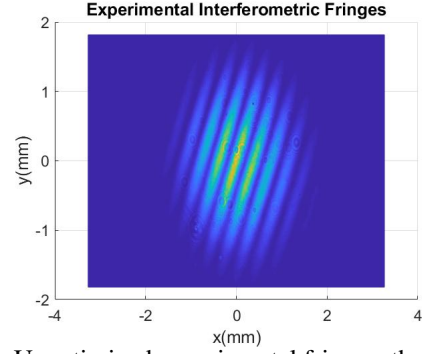
$$E_{tot}(x, y) = 2E_0 \cos\left(k\frac{\alpha - \beta}{2}x + \frac{1}{2}\Delta\varphi(x, y)\right) \cos(\omega t - \Theta) \quad (7)$$

Where  $\Delta\varphi(x, y) = \varphi(x, y) - \varphi_0$  and  $\Theta$  contains all remaining contribution of the sum. The interferometric pattern, typically measured, is an intensity profile, so we may prefer to find the  $I(x, y)$  intensity profile, proportional to time average of the modulus square  $I \propto \langle |E|^2 \rangle$ .

Therefore,

$$I(x, y) = I_0 \cos^2(k(\alpha - \beta)x + \Delta\varphi(x, y)) \quad (8)$$

Where  $\alpha - \beta$  is, indeed,  $\gamma$  the angle between the two wave vectors. The result is clear, a part from a  $\Delta\varphi(x, y)$ , that will be the later main matter, we have the generation of interferometric fringes extended in  $y$ -direction with an  $x$ -spacial frequency given by  $\nu_x = \frac{\lambda}{\gamma}$ .



**Figure 3.** Unoptimized experimental fringes, the  $k$ -vectors do not lie in  $(x, z)$ ,  $\Delta\varphi(x, y) = 0$

## 1.2 Refractive index profile and Phase variation

At this point, we may go further and discuss the effect of a phase front distortion due to the presence of a non-absorbent obstacle, the 3D gas jet.

We want to solve the following problem:

*Given a certain  $n(x, y, z)$  refractive index how a plane monochromatic wave that passes through is affected ?*

Starting from Helmholtz equation for a linearly polarized wave:

$$\nabla^2 E(x, y, z) + n^2(x, y, z) \frac{\omega^2}{c^2} E(x, y, z) = 0 \quad (9)$$

Inserting as an ansatz the plane wave, with  $k$   $z$ -oriented,  $E(x, y, z) = E_0 e^{ikz + i\varphi(x, y, z)}$  we solve for  $\varphi(x, y, z)$ . If we neglect the phase variation among  $x$  and  $y$ , as well as all second derivatives, we end up with the following equation:

$$-\left(\frac{\partial\varphi(x, y, z)}{\partial z}\right)^2 + n(x, y, z)^2 \frac{\omega^2}{c^2} = 0 \quad (10)$$

Therefore, under the previous hypothesis, the phase accumulated by our monochromatic waves through such a medium is:

$$\varphi(x, y, z) = \int_{-\infty}^z n(x, y, z') k_0 dz' \quad (11)$$

This expression can be solved for a generic beam profile, however, it is not particularly comfortable to identify a good  $z$  where I have to stop.

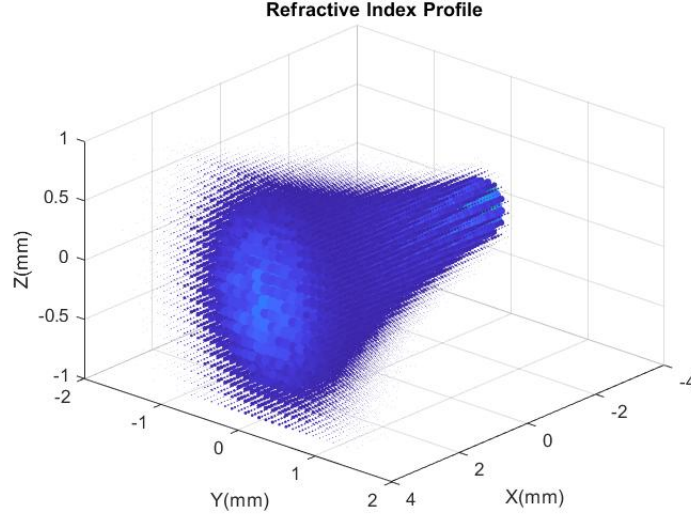
Therefore, we're going to consider the  $\Delta\varphi(x, y, z)$  and the related  $\Delta n(x, y, z)$ , between the two optical paths.

This allows us to write

$$\Delta\varphi(x, y) = \int_{-\infty}^{+\infty} \Delta n(x, y, z) k_0 dz \quad (12)$$

Note the lack of dependence on the  $z$ -axis of  $\Delta\varphi$ . Indeed, once we've passed through the gas jet, ideally, no further phase offset will appears between the two optical path.

To run a simulation we need to build up a 3D profile index, defining the profile index of a gas jet of molecules, or atoms, is not a trivial problem at all. The refractive index is somehow



**Figure 4.** Simulated refractive index profile with additional noise embedded

related to the density of the gas in space. However, the actual profile is a fluid-dynamic problem and, a priori, we should spend many pages to define a reasonable profile, see<sup>[1]</sup>.

To simplify the problem we can imagine that our jet is a composition of 2D super-gaussian profile that, keeping the total area constant, expands with a certain slope.

In the following simulations, I have assumed a profile of the form:

$$n(x, y, z) = n_0 \sqrt{\frac{\pi}{R(x)}} e^{-\frac{(y^2+z^2)}{2R(x)}} \quad R(x) = e^{\alpha x} + R_0$$

Where  $\alpha$ ,  $R_0$  and  $n_0$  are arbitrarily parameters, chosen to fit the wanted spacial propriety of the gas jet.  $n_0$  has a particularly relevant meaning, its magnitude is function of the gas density and, it can be shown that a more dense gas produces a bigger phase offset.

A simulated profile is shown in Figure 4.

### 1.3 Phase recovery via Fourier Transform

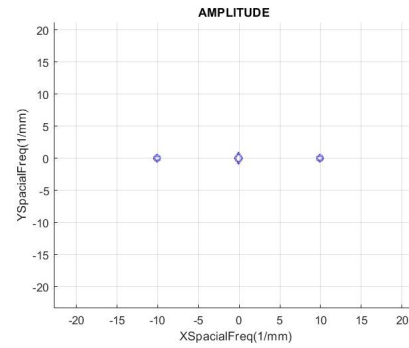
The problem is very general, from a certain interferometric pattern of the form (8), we want to recover the concealed phase information  $\Delta\phi(x, y)$ , generated by a certain refractive index profile through (12). A widely established method is to reconstruct the phase via 2D-Fourier analysis. Consider a certain generalized interference fringes pattern:

$$I(x, y) = I_A(x, y) + I_B(x, y) \cos(2\pi v_{x0}x + \Delta\phi(x, y)) \quad (13)$$

Where  $I_A(x, y)$  and  $I_B(x, y)$  represent a slowly varying envelope, e.g. a gaussian modulation of the intensity. At this point we can perform the Fourier transform of (13):

$$\begin{aligned} \tilde{I}(v_x, v_y) = & \tilde{I}_B(v_x, v_y) * \left[ \frac{1}{2} \delta(v_y, v_x + v_{x0}) * \mathcal{F}(e^{-i\Delta\phi(x, y)}) + \right. \\ & \left. + \frac{1}{2} \delta(v_y, v_x - v_{x0}) * \mathcal{F}(e^{i\Delta\phi(x, y)}) \right] + \tilde{I}_A(v_x, v_y) \end{aligned}$$

In practical terms it means that in the spectral domain we will have two sharp peaks at respectively,  $(v_x, 0)$  and  $(-v_x, 0)$ , that encodes the phase information, and a central peak at zero frequency.



**Figure 5.** Simulated spectral domain amplitude, 2D-Fourier transform.

At this point, if we are able to filter only one of the two lateral peaks we remain with:

$$\tilde{I}_f(v_x, v_y) = \tilde{I}_B(v_x, v_y) * \left[ \frac{1}{2} \delta(v_y, v_x - v_{x0}) * \mathcal{F}(e^{i\Delta\phi(x, y)}) \right] \quad (14)$$

Now, if we anti-transform this term, we finally achieve:

$$I_f(x, y) = \frac{1}{2} I_B(x, y) e^{i2\pi v_{x0}x + i\Delta\phi(x, y)} \quad (15)$$

The needed information now is encoded in the phase of this complex quantity. Once we have subtracted the constant slope given by the  $v_{x0}$  modulation, we can, finally, recover the aimed  $\Delta\phi(x, y)$ . To go from  $\Delta\phi(x, y)$  to  $n(x, y, z)$  is a common inverse problem, that won't be discussed.

## 2. Matlab Simulated phase Reconstruction

### 2.1 Introduction to the simulated fringes:

In this part, I am going to present, in details, the simple code that can be implemented to accomplish the task. In the next part real experimental profiles will be analyzed and discussed.<sup>1</sup>

Since we are dealing with a simulation, an index profile has to be created. A profile index is a scalar field,  $n(x, y, z)$ , so we need to create a 3D meshgrid to associate to each point of the space a certain value.<sup>2</sup>

At this point we only need to simulate the phase off-set, this can be, easily, done with a numerical integration. Finally, we can create the interference fringes profile, of the form (8).

To have a general understanding of the effect of such an index profile on the interference fringes, two simulation have been performed with and without the presence of the gas jet.

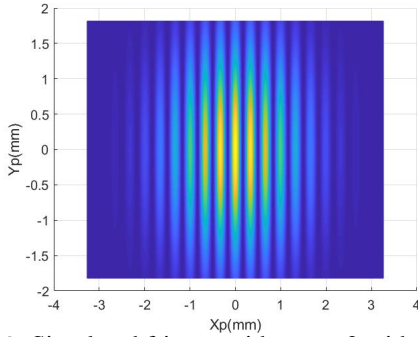


Figure 6. Simulated fringes with  $v_{x0} = 3$  without Gas Jet

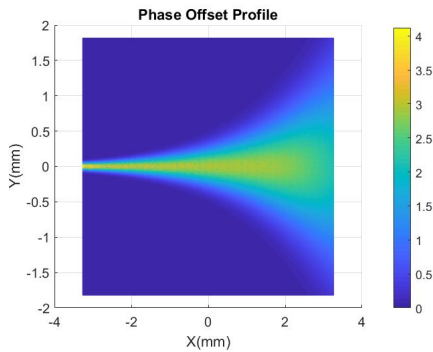


Figure 7. Phase Offset induced by the gas jet profile.

The gas jet is modifying the relative position of the fringes, in the following way:

<sup>1</sup>You can find the Matlab code and related simulation following the link at <https://www.youtube.com/channel/UCbuWPg3AJ1X3fKXo6AbJfSA> in the information section.

<sup>2</sup>Obviously we are assuming that a description with  $n(x, y, z)$  instead of the tensorial  $\chi$  is reasonable for our medium.

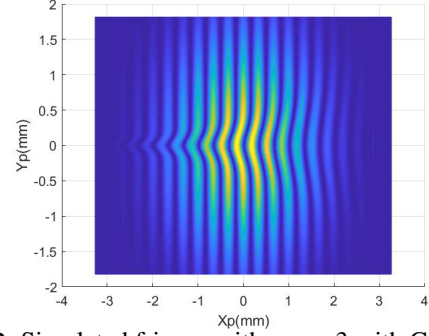


Figure 8. Simulated fringes with  $v_{x0} = 3$  with Gas Jet, you can observe a stronger fringes bending on the left, where the gas jet is more intense.

- Fixed  $\bar{y}$ , we have that the intensity profile is shifted with respect its unperturbed position, Figure 6 of a quantity that depends on the local offset  $\Delta\phi(x, \bar{y})$
- Fixed  $\bar{x}$ , the magnitude of the shift decrease following the phase offset profile  $\Delta\phi(\bar{x}, y)$ .

The next step is to analyze the interference fringes with Fourier analysis. Unfortunately, in a simulation, as well in a real measurement, we do not have access to all the continuum of space, but we have both a discrete and limited image<sup>3</sup>, like Figure 8. Therefore, we must use discrete Fourier transform. Let's focus on the 1D problem definition, then we will perform a row by row analysis of the 2D intensity profile to recover the phase. This alternative approach<sup>4</sup>, as I will show works extremely fine, however it requires that the fringes **must not** have any modulation on y-axis, i.e.  $v_{y0} = 0$ .

Indeed, the most general form of (8) should be:

$$I(x, y) = I_0 \cos^2(v_{x0}x + v_{y0}y + \Delta\phi(x, y)) \quad (16)$$

As I will mention later, this is not a big problem with a real experiment, since we can manually (or automatically) fix the fringes orientation to have an almost negligible y-modulation. Therefore, from now on, only  $v_{y0} = 0$  fringes will be discussed.

### 2.2 1D Phase recovery, main idea

The first problem, one may encounter, is the definition of the frequency axis and the minimum frequency step detectable. Given  $N$  the  $x$ -axis size and  $dx$  the incremental step. Then, the frequency axis will reasonable extended to a maximum frequency that is the Nyquist frequency  $f_{NQ} = \frac{1}{2dx}$  with an incremental step of  $\delta f = \frac{1}{Nd_x}$ , see for details [2].

At this point, the row by row Fourier analysis of the intensity profile should be performed.

<sup>3</sup>Although we can always, extend the image dimension manually, by adding zeros at the edges.

<sup>4</sup>Alternative to a 2D Fourier analysis.

Can we reconstruct, arbitrarily, a generic Gaussian phase profile? For instance, can we recover a profile with spacial dimension  $\Delta x = 0.1$  mm, does it make any difference to set a certain  $v_x$ ? How does the filtering of the left peak work? To answer, precisely to this answer we must calculate exactly  $\mathcal{F}(e^{i\Delta\phi(x,y)})$  and then comments the bandwidth occupied varying  $n_0$  the refractive index magnitude and the minimum spacial width of simulated gas jet. It is, indeed, clear that to recover the phase we should not filter-out relevant components of the left peak, but since a peak at DC is always present, we are limited by the superposition of the left peak with the central one. Therefore, the fringes spacial frequency is needed to be properly set both in the experiment and in practice. The procedure in the 1D case is simple and do not require special intervention, apart of a proper generation of the frequency axis and, in the most general case, by cutting the edges of the reconstructed profile<sup>5</sup> The 1D phase recovery works as follow, you enter a certain intensity profile:

$$I(x) = I_0(x) \cos(v_x x + \Delta\phi(x)) \quad (17)$$

Then you perform the Fourier transform, and you obtain three peaks. With a super-Gaussian<sup>6</sup> function, properly defined, you perform a cross-correlation with the right part of the spectrum, this allows to find automatically the  $v_x$ .

At this point you filter this peak with another super-Gaussian function centered in the previously determined  $v_x$ , this will modify a bit the spectrum but it will help to reduce effect that a simple sharp cut will eventually introduce. For design choices, I have decided to build up a reconstruction program that perform a row by row analysis of the 2D patter. In the following part I'm going to report the result of the simulations.

### 2.3 2D Phase reconstruction simulation

Playing with the refractive index many profiles have been generated, all have been reconstructed successfully. This has been achieved only within a certain range of fringes frequency, that is fixed by the profile proprieties.

The final test, that confirmed the potentiality of the code and, ultimately, have been thought to be a reconstruction of a phase profile with a sharp discontinuity. The simulations allow to have an idea of the possible problem in the reconstruction of the real life measurements, and fixes the requirement of the experimental setup. Clearly, we expect that many non idealities will interfere with the reconstruction algorithm. Therefore, the code and has been tested in different condition, progressively adding noise on the phase profile. This brings to a parameter optimization, that have the only purpose to understand the range of reconstruction capability of this simple algorithm. The obtained "optimal parameters" values have poor experimental validity since, as the gas jet literature confirm, the

<sup>5</sup>You can significantly reduce this problem if you extend the simulated fringes adding zeros. This will increase the frequency resolution as well as removing edges problem.

<sup>6</sup>Take a Gaussian and elevate it to a certain power 5-20, this gives you a very sharp filter but continuous.

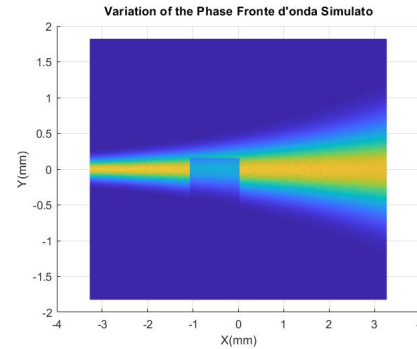


Figure 9. Simulated discontinuous phase profile

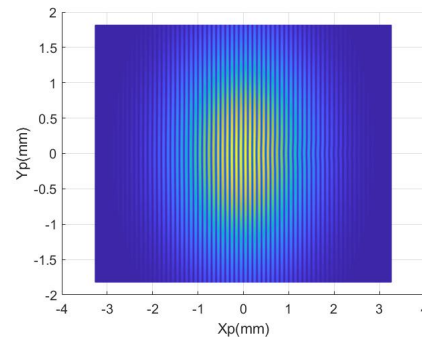


Figure 10. Simulated fringes with  $v_{x0} = 10$  with additional Gaussian envelope, this interferometric profile embeds the previous phase profile.

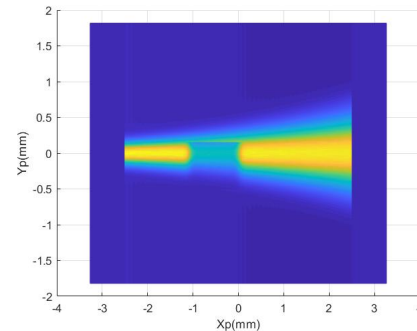


Figure 11. Reconstructed phase profile by interferometric fringes reconstruction

assumption of diffusive Gaussian profile for the jet does not reflect the reality, at all. However, the give us an insight on the possible problems during reconstruction.



### 3. Experimental Setup and Gas Jet Characterization Results

Implemented the reconstruction code it was time to perform some real measurement to characterize the gas jet. This gas jet is going to be used in the Attosecond Research Center at Politecnico of Milano to see biomolecules attosecond dynamics. The characterization has been made in air, at first, to both being sure that no unexpected behavior shows up<sup>7</sup>, and second to evaluate the accordance with the literature, this can be seen as another test for the reconstruction code. At last a full vacuum characterization has also been performed.

A fundamental question, totally unrelated to the main matter of discussion is *Why you need to characterize the gas jet?*. And even more important *If you need to characterize the gas jet for an experiment, then where the experiment is going to be performed and which are the parameter you need to take under control?*. These are very good question that have been taken into account during the whole design. However, since the aim of this paper is to report a characterization we will omit this part.

#### 3.1 Air characterization

With an outside pressure of 1 bar the gas jet was loaded with compress air. To perform a two waves interference, as reported at the beginning of the paper, a Mach-Zehnder interferometer has been build.

Gas Jet Interferometer

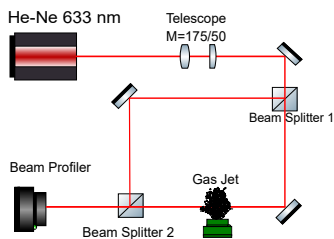


Figure 12. Experimental setup sketch, Mach-Zehnder interferometer

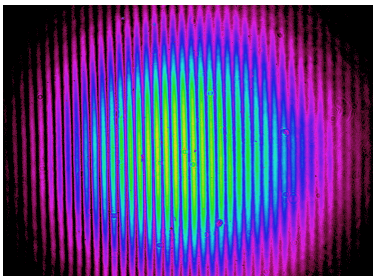


Figure 13. Experimental beam profile unfiltered acquisition

The realization of a Galileo telescope was an experimental

<sup>7</sup>Maybe due to mechanical damages of the different mechanical components

need dictate by the inadequate initial beam size with respect the gas jet aperture diameter,  $d = 1$  mm. To have a good matching the beam size has been brought to a diameter of roughly 4 mm. The final interference pattern was acquired by a beam profiler. The bare acquisition have brought many problems, mainly due to camera defects, and sensitivity of the acquisition system on the fringes pattern modification, e.g. the pixel resolution. The holes in the 2D image brings several issue, they can be removed in principle with a differential measurement, i.e. one measures without the gas jet and subtract the reconstructed profile from each gas jet measurement. However, this is not always possible, as it is clear from figure 8, the gas jet alters the fringes pattern, therefore the "holes" contribution is not constant. To face this problem smoothing and filtering have also been introduced before and after the Phase Recovery algorithm. At a temperature of  $T = 295$  K many measurements have been performed, as close as possible to the gas jet output nozzle, in this way an higher signal could be measured. The compress air jet was varied between 3-5 bar of absolute pressure.

Below we report the results:

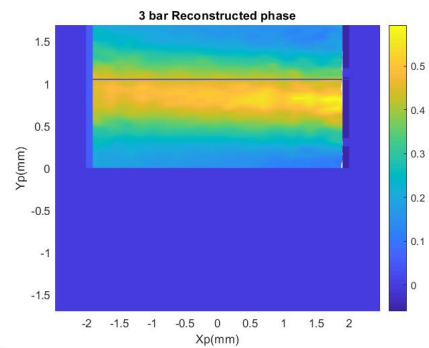


Figure 14. 3 bar profile reconstruction, in air

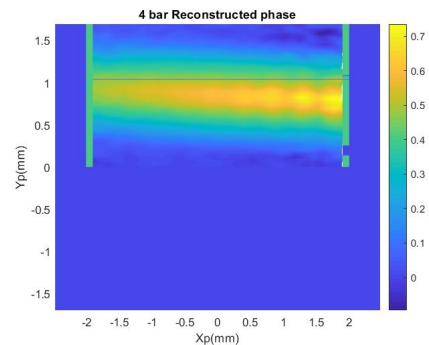
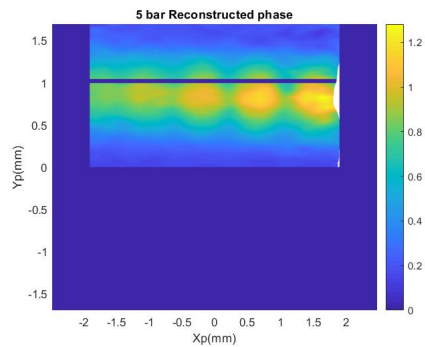


Figure 15. 4 bar profile reconstruction, in air

The experimental result clearly are against the simulation assumption of gaussian profile. Fortunately, they are quite in agree with the result reported in literature. Before proceeding further, we want to stress the fact that the reported pressure is



**Figure 16.** 5 bar profile reconstruction, in air

referred to the one injected in the gas jet. For fluid-dynamic reasons this pressure is very different with respect the one at the nozzle output.

What can be observed is the formation of what are called "Mach diamonds" or "Shock diamonds", these are a peculiar turbulence states induced by a supersonic behavior of the jet. Without entering into details on the effect, we just say that this regime is induced by a fight between the will of the gas, below atmospheric pressure, to expand and the environment pressure that wants, instead, to compress it. At the beginning the atmospheric compression wins, due to the lower pressure of the gas, however, when the gas is too much compressed, its pressure, and its local density<sup>8</sup>, increases and the expansion takes the control back.

What we can infer, at the end of this stage of measurement, is:

- The gas jet behave accordingly to the experimental expectation, shock diamonds can be seen in all the measurement.
- No huge mechanical defect alters neither the output flow direction, nor the typical fluid-dynamic behavior.
- The variation of pressure, therefore, the particle density at the nozzle, as expected, influence the magnitude of the phase offset.
- The final experiment, that is going to be conducted in vacuum, will show a completely different behavior. It is indeed true that no shock diamonds, given their nature, will be present. A characterization in this different environmental condition is therefore needed.

### 3.2 Vacuum characterization

#### Conclusion

#### Acknowledgments

#### References

- [1] Lukas Lessmann,  
*Investigations on gas jet beam profiles with a Mach-Zehnder interferometer.*  
Master Thesis at Munster, 2016.

<sup>8</sup>This is the reason why we have a more intense